ON THE CONSERVATION OF VORTICITY IN THE MAGNETOHYDRODYNAMICS OF PLASMA WITH MANY COMPONENTS

(O SOKHRANENII VIKHRIA V MNOGOKOMPONENTNOI MAGNITNOI GIDRODINAMIKE)

PMM Vol.24, No.6, 1960, p. 1123

N.V. SALTANOV
(Sukhumi)

(Received April 8, 1960)

The system of equations for the hydrodynamic approximation of an ideal plasma consisting of N kinds of ions ($k = 1, \ldots, N$) takes the form [1-4]

$$\frac{\partial \mathbf{v}_k}{\partial t} + \nabla \left(\frac{v_k^2}{2} + \int \frac{dp_k}{n_k m_k} + F_k \right) = \mathbf{v}_k \times \left(\operatorname{rot} \mathbf{v}_k + \frac{\mu e_k}{c m_k} \mathbf{H} \right) + \frac{e_k}{m_k} \mathbf{E}$$

$$\frac{\partial n_k}{\partial t} + \operatorname{div} n_k \mathbf{v}_k = 0, \qquad \operatorname{div} \mathbf{H} = 0, \qquad \operatorname{div} \mathbf{E} = \frac{4\pi}{\varepsilon} \sum_{j=1}^{N} e_j n_j \tag{1}$$

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c} \sum_{j=1}^{N} e_j n_j v_j + \frac{\varepsilon}{c} \frac{\partial \mathbf{E}}{\partial t}, \qquad \operatorname{rot} \mathbf{E} = -\frac{\mu}{c} \frac{\partial \mathbf{H}}{\partial t}$$

The first two equations are the momentum and continuity equations for the ions of species k, and the rest are Maxwell's equations. The inertial term in the momentum equation is given in the form of I.S. Gromeka; $\mathbf{H}$ and $\mathbf{E}$ denote the intensity of the magnetic and electric fields, μ the magnetic permeability, ϵ the dielectric constant and c the speed of light. F_k represents the potential of external forces, $\mathbf{V}_k$ the velocity, p_k the partial pressure, n_k the number of particles per unit volume, e_k and m_k the charge and the mass, all pertaining to species k.

The plasma is considered either as a "cold" ($p_k = 0$) or an incompressible ($n_k = \text{const}$) or, again, a barotropic ($p_k = p_k(n_k)$) medium.

When the curl operator is applied to the momentum equation and use is made of the equation for the electric induction, there results

$$\frac{\partial \mathbf{\Omega}_k}{\partial t} = \operatorname{rot} \mathbf{v}_k \times \mathbf{\Omega}_k, \qquad \mathbf{\Omega}_k \equiv \operatorname{rot} \mathbf{v}_k + \frac{\mu e_k}{c m_k} \mathbf{H} \tag{2}$$

The conservation of the partial vorticity Ω_k is analogous to the conservation of vorticity in ordinary hydrodynamics as shown by Helmholtz [4]. Hence, the vortex lines corresponding to Ω_k are "glued" to the particles of the k species, and the flux of the partial vorticity Ω_k across an arbitrary material surface, made up of particles of species k, is conserved. Furthermore, the circulation of the vector

$$\mathbf{v}_k + (\mu e_k / c m_k)\,\mathbf{A}\,(\mathbf{H} = \operatorname{rot} A)$$

along arbitrary closed contours, made up of particles of species k, is also conserved. We remark that the conservation of the partial vorticity ceases as a result of viscous forces and collisions between particles of different species.

For the cold electron-ion plasma the vorticity-conservation laws were noted in [1]. In [2], where the plasma was described by a system of magnetohydrodynamic equations for a binary (electron-ion) medium, the conservation of the partial vorticity for the ionic component was brought out.

For the cases of the steady two-parameter and the steady vortical plasma motions the integral form of the conservation of the partial vorticity was noted in [3].

In conclusion, the author wishes to express his deep appreciation to V.S. Tkalitch for his interest and valuable advice.

BIBLIOGRAPHY

1. Ferraro, V.C.A., Hydromagnetic waves in rarefied ionized gas and galactic magnetic fields. *Proc. Roy. Soc.* A233, No. 1194, 1955.

2. Braginskii, S.I., Ob odnom kriterii primenimosti uravnenii magnitnoi gidrodinamiki k plazme (On a criterion of applicability of the magnetohydrodynamic equations to a plasma). Symposium: *Problems of Magnetohydrodynamics and Plasma Dynamics*, p. 67. Riga, 1959.

3. Tkalitch, V.S., Preobrazovanie sistemy uravnenii gidrodinamicheskogo priblizhenia plasmy (Transformation of the system of equations of the magnetohydrodynamic approximation for plasma). *Izv. Akad. Nauk SSSR, OTN, Mekhanika i mashinostroenie* No. 5, 1959.

4. Kochin, N.E., Kibel, I.A. and Roze, N.V., *Teoreticheskaia gidromekhanika (Theoretical Hydrodynamics)*, Vol. 1. GITTL, 1948.

Translated by M.V.M.